NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 35BAMS	**LEVEL:** 6
COURSE CODE: NUM701S	**COURSE NAME:** NUMERICAL METHODS 1
SESSION: JUNE 2022	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr S.N. NEOSSI NGUETCHUE
MODERATOR:	Prof S.S. MOTSA

INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations. All numerical results must be given using 4 decimals where necessary unless mentioned otherwise.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 2 PAGES (Including this front page)

Attachments

None

Problem 1 [30 marks]

1.1. If $f \in C^{n+1}[a,b]$, prove that for any points x and c in $[a,b]$, we have [12]

$$f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k + R_n(x) \quad \text{where} \quad R_n(x) = \frac{1}{n!}\int_c^x f^{(n+1)}(t)(x-t)^n dt$$

[Hint: use integration by parts $\int udv = uv - \int vdu$ with appropriate choice of u and v.]

1.2. Consider $f(x) = -\frac{1}{2}x^2 + 3x - 4 = 0,\ x \in [3.5, 4.5]$.
Use Newton's method to approximate the root of the above equation after three iterations. [4]

1.3. The equation $x = g(x) = (x^2-1)/3$ has a root in $[-1,1]$.

1.3.1. State the fixed-point Theorem. [4]

1.3.2. Prove that the sequence $(x_k)_{k\in\mathbb{N}}$ with $x_{k+1} = g(x_k)$ converges to the fixed-point of the equation given above in **1.3.** for any choice of $x_0 \in [-1,1]$. [10]

Problem 2. [40 marks]

2.1. Write down in details the formulae of the Lagrange and Newton's form of the polynomial that interpolates the set of data points $(x_0, f(x_0)), (x_1, f(x_1)), \ldots, (x_n, f(x_n))$. [7]

2.2. Use the results in **2.1.** to determine the Lagrange and Newton's form of the polynomial that interpolates the set of data points $(0,1), (1,6)$ and $(2,17)$. [18]

2.3. Establish the error term for the rule: [15]

$$f'''(x) \approx \frac{1}{2h^3}[3f(x+h) - 10f(x) + 12f(x-h) - 6f(x-2h) + f(x-3h)]$$

Problem 3. [30 marks]

Given the IVP

$$y' = ty + y + t^2, \quad y(0) = 2. \tag{1}$$

3.1 Write down in details the fouth-order Runge-Kutta (RK4) algorithm to solve the specific IVP given by Eq. (1). [10]

3.2 Given the table below, use the result of question **3.1** to compute the missing values. [20]

k	t_k	k_1	k_2	k_3	k_4	y_k
1	0.08	2	2.1648		2.35403	2.17369
2	0.16		2.55439		2.78496	
3		2.78488	3.0281			2.62174
4		3.30856		3.61874	3.94524	
5	0.4		4.30325		4.71963	

TOTAL: 100 marks

God bless you !!!